

Rav's Beautiful Ratio: An Excursion into Aesthetics

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The Talmud Sage Rav teaches that when tying *tsitsit* (ritual tassels) there is both an upper limit and a lower limit for the amount of braiding, as well as an ideal—"beautiful"—amount given in the form of a ratio of braided to unbraided sections. Embedded within these dry details lies the key to understanding the profound nature of aesthetics within Jewish thought. To what extent does Jewish law demand this ratio to be met and why? If the sole reason is simply "beauty," of what concern is aesthetics to halakhic imperative?

By answering these questions, an appreciation of the depth of Rav's teaching can be obtained—an appreciation that touches upon the very essence of the human quest to encounter the Divine. This excursion into aesthetics delves into the number *phi*, known as the Divine Proportion, as well as the positions of Immanuel Kant and Rabbi Joseph B. Soloveitchik.

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Introduction

In Numbers 15:38 and Deuteronomy 22:12, the Torah commands that *tsitsit* (tassels) be placed on four of the corners of a four- (or more) cornered garment. The tassels are to serve as a reminder to fulfill all the commandments of the Torah (Numbers 15:40). They are also seen to be an insignia of the King of kings, worn by His faithful servants (*Mahzor Vitri* 409).

Regarding how the tassels are made, the Talmud provides sparse few statements, thus leading to a plethora of interpretations. We shall use the description given by Maimonides in the twelfth century CE in his monumental compendium of Jewish law, the *Mishneh Torah*:

The tsitsit tassel is made by placing four strings through a hole in the corner of the garment and folding them in half, thus making a tassel of eight strings. One of these eight strings is *tekhelet* [blue] and the remaining seven are white. The blue strand and one of the white strands are used to wrap around the hanging strands, such that one starts with a white wrap and ends with a white wrap, the rest of the wraps being all blue. The wrappings are to be organized of between seven to thirteen *hulyot* [links], wherein each *hulyah* comprises three wraps. The wrapped section of the tsitsit is referred to collectively as the *gdil* [braid], and the free hanging section of the strings is referred to as the *anaf* [fringe] (*Mishneh Torah, Hilkhhot Tsitsit* 1:7-8).

The photo on the inside back cover of this volume illustrates tsitsit tied according to Maimonides' description. The ratio for the proportion of braided string versus the free hanging fringe was defined in the second century CE by the great Babylonian Talmud Sage Rav as follows:

If the greater portion of the tekhelet [i.e., tsitsit tassel]¹ was braided, it is valid; and even if only a single *hulyah* [out of the three wraps]² was braided, it is valid; and the way to beautify [*v'noyei*] the tekhelet is one-third *gdil* [braided] and two-thirds *anaf* [fringe] (Talmud *Menahot* 39a).

Rav teaches that when tying tsitsit there is both an upper limit³ and a lower limit for the amount of braiding, as well as an ideal—"beautiful"—amount given in the form of a ratio of braided to unbraided sections. Embedded within these dry details lies the key to understanding the profound nature of aesthetics in Jewish thought. We can appreciate the depth of Rav's teaching by answering the following two questions:

1. To what extent does the *halakhah* (Jewish law) demand this ratio and why?
2. If the sole reason is simply "beauty," of what concern is aesthetics to halakhic imperative?

Halakhah

The following halakhic overview is based on Rabbi Shlomoh Taitelbaum's article in Hebrew, "*Shnei Shlish*."⁴

Commenting on the word *v'noyei* (to beautify) in Rav's definition, Rashi explains that Rav's ratio of one-third braid and two-thirds fringe is the principal [*ikar*] way to fulfill the commandment of tsitsit. Rabbeinu Tam, commenting on Talmud *Bkhorot* 39a, (s.v. *kama*), states that the ratio of one-third braid to two-thirds fringe optimizes (*hidur*) the commandment to tie tsitsit.

In *Hilkhot Tsitsit* 1:8, Maimonides echoes Rav's statement that the way to beautify tsitsit is to make them according to the thirds ratio. In *Hilkhot Tsitsit* 1:9, Maimonides goes on to explain that when there are no tekhelet strings available, maintaining the thirds ratio is not imperative.⁵ Rabbi Yosef Karo, in his commentary *Kesef Mishnah*, picks up on the implication of this last statement by Maimonides and explains his language to mean that when tekhelet strings *are* available, one *must* employ Rav's ratio.⁶

The Tur on *Orah Hayyim* 11 is of the opinion that one must maintain Rav's ratio apparently, regardless of the availability of tekhelet. The Tur cites Baal Ha'Tur, who rules that though any ratio may be used, the principal (*ikar*) method of fulfilling the commandment is according to Rav's ratio. Rama, in his *Darkhei Moshe* on the Tur, quotes Maimonides' imperative that Rav's ratio must be used.

Rabbi Yosef Karo in his *Beit Yosef* (*Orah Hayyim* 11:4) discusses the requirement to maintain Rav's ratio and specifies the minimum lengths to be used for the ratio as borne out from the Sifri. In his *Shulhan Arukh*, Rabbi Karo simplifies the matter and writes that the *minimum* required lengths for "the braided section is four thumb-breadths and the fringe section, eight thumb-breadths." Rama comments on this that these lengths can be increased as long as Rav's ratio is maintained.

Hazon Ish (3:1) and Maharshal (on Smag, positive commandment 26) are of the opinion that the fringe can be lengthened without modifying the braided section, and this would still "beautify" the tsitsit. *Mishnah Brurah* (*Orah Hayyim* 11:69) explains that if the tsitsit has already been tied in this manner, then all the authorities will accept it as kosher;⁷ conversely,

if one is setting out to tie his tsitsit he must use Rav's ratio.⁸

To summarize, with the exception of the opinions of Hazon Ish and Maharshah, the weight of halakhic opinion rests squarely with Rav's ratio and his absolute requirement. It should be noted that the retroactive leniency noted by the *Mishnah Brurah* is applicable only in urgent circumstances,⁹ for if the tsitsit lacks the proper ratio, it is relatively simple to correct this by either cutting the fringe (anaf) or modifying the braid (gdil) by adding or removing winds and knots. If none of these options is viable¹⁰—it is not difficult to simply replace the strings in question. Thus, the leniency mentioned by the *Mishnah Brurah* is at most of ephemeral efficacy and cannot be relied on for any extended period of time.

Beauty

The overwhelming support for obligating Rav's ratio begs the question: Why? Why is maintaining Rav's ratio so important? Rav himself answers this question in one word: *noy* (beauty). But if the reason is beauty, how can one be obligated to another's definition? Is not beauty, as the eighteenth-century proverb goes, in the eyes of the beholder?

Antimony

This question brings us into the thick of a great philosophical debate known as "the antimony of taste." Antimony refers to a contradiction between two rational statements arrived at through logical reasoning. The antimony regarding taste considers whether aesthetic value is subjective or objective. On one hand, most people have a feeling about what they find pleasing; on the other hand, without universal criteria for making aesthetic judgments, appreciation of the aesthetic is reduced to the level of mere gustatory taste.¹¹

There are important arguments to be made for both sides of this debate, however, perhaps the most amenable solution lies in reconciliation.¹² That is, beauty can be defined both as something which is appreciated uniquely by the subject, the object of which nevertheless admits of some aesthetic quality.¹³ Beauty, then, is the integration of subjective aesthetic appreciation and objective aesthetic quality.

Phi

By definition, every person determines for him or herself the subjective part of the equation. But what defines objective beauty? The fifth-century BCE Greek philosopher Pythagoras argued that mathematics quantified the beautiful.¹⁴ The eighteenth-century German philosopher Immanuel Kant stated that "Art can only be called beautiful if we are conscious of it as Art while yet it looks like Nature."¹⁵ Combining these two concepts, mathematical structures found in nature could be said to represent universally recognized, objective beauty.¹⁶

Such a phenomenon is found in the extraordinary irrational number known as *phi* (the Greek letter Φ).¹⁷ In his book "The Golden Ratio," Mario Livio writes of the number *phi*:

The history of art shows that in the long search for an elusive canon or "perfect" proportion, one that would somehow automatically confer aesthetically pleasing qualities on all works of art, the Golden Ratio has proven to be the most enduring.¹⁸

The number *phi*, though known of earlier,¹⁹ was formally defined in the third century BCE by Euclid in his fundamental opus *Elements*.²⁰ The number emerges from a simple geometric division of a line into two unequal parts, such that the ratio of the longer segment to the shorter segment is equal to the ratio of the total length to the longer segment. Assume the following line composed of a short line segment *s* and a long line segment *l*:

|----- s -----|----- l -----|

The irrational number *phi* (1.618...) is realized when:

$$(l+s) / l = l/s$$

Beyond expressing this simple ratio, *phi* is intimately associated with the Fibonacci Series. The Fibonacci Series was made famous by the thirteenth century Italian mathematician Leonardo of Pisa. In his *Liber Abaci*, Leonardo of Pisa describes a series of numbers in which the next number is the sum of the preceding two numbers. If a Fibonacci series starts with the two numbers 1 and 1, then the next number is 2, the number after that is 3, and after that is 5, thus giving the series:

1, 1, 2, 3, 5, 8, 13...

In 1611 the astronomer Johannes Kepler discovered that if we denote the n^{th} number in the series as F_n and its successor as F_{n+1} then F_{n+1} / F_n provides closer and closer approximations of phi as n increases.

Kepler's find is also given expression in the geometric relationship between the Fibonacci series and phi. Stacking squares with sides measuring F_n in succession (for example, where $n = \{1, \dots, 7\}$, as shown below), the rectangle formed by the combination of n adjacent square will have its longer side measure F_{n+1} and its shorter side measure F_n — the ratio of the sides being F_{n+1} / F_n . Consequently, Kepler's numerical discovery also applies geometrically. This means that as n increases, the rectangle formed has sides the ratio of which exhibits a closer and closer approximation of the Golden Ratio. A rectangle wherein the ratio of the longer side (l) to the shorter side (s) is equal to phi (i.e., $l/s = \Phi$) is known as a Golden Rectangle (illustrated in Figure 1).

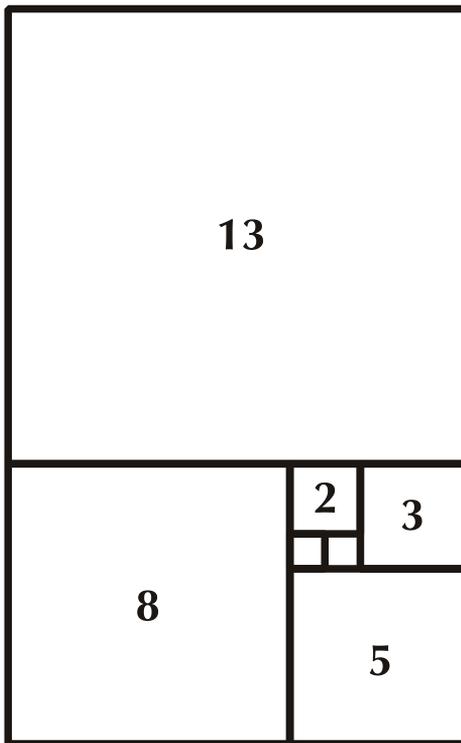


Figure 1. A Golden Rectangle

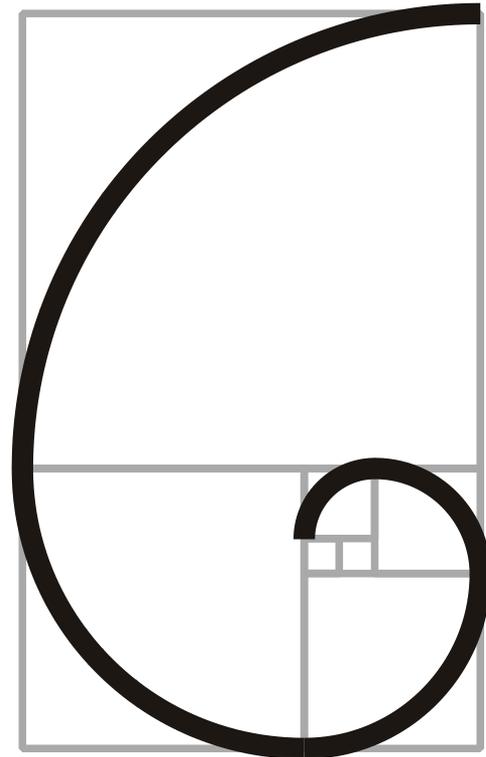


Figure 2. Spira mirabilis, a logarithmic spiral

Furthermore, if we were to draw quarter circles from the diagonal corners of each of these squares and connect them, as shown in Figure 2, we would achieve a “logarithmic spiral.”²¹ Due to its wonderful property of never changing its shape when it increases in size this spiral was dubbed the *spira mirabilis* (wonderful spiral) by the seventeenth-century Swiss mathematician Jacques Bernoulli.²² The logarithmic spiral is thus closely connected to the Golden Ratio.

Another important expression of phi is in its description of the Golden Angle. This angle is achieved by a 0.618 ($1/\Phi$) turn around an axis. Calculating this turn in degrees, we simply multiply $1/\Phi$ by 360° , such that 360° times 0.618 gives 222.5° and its complementary angle of 137.5° (i.e., 360° minus 222.5°). This angle figures extensively in the study of growth patterns of plants, known in botany as phyllotaxis, where it is found to be the most efficient angle of growth around an axis. Phyllotaxis also shows that not only do leaves grow around a stem using this angle, and not only do the florets on the head of a sunflower grow accordingly, but they also grow in interleaving logarithmic spirals which number according to Fibonacci pairs. Figure 3 shows a typical growth pattern where the clockwise spirals number 13 and the counterclockwise spirals number 21.²³

Thus, the number phi in its simplest form represents a proportion

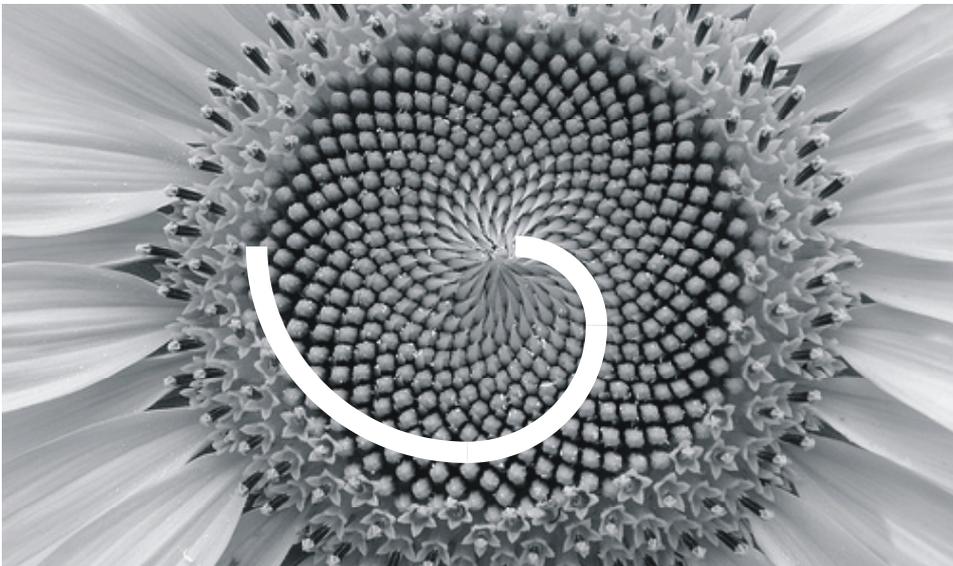


Figure 3. Typical growth pattern of a plant

equaling 1.618..., giving us the Golden Section of a line segment, as well as the proportions of the Golden Rectangle. Phi is also expressed in the logarithmic spiral and is the source of the Golden Angle, as well as many other mathematical forms too numerous to mention.²⁴ Phi, in all these various forms, has been found so ubiquitously in nature that it has been labeled the Divine Proportion.²⁵ In addition to its botanical manifestations, phi is found to describe the attack pattern of the Peregrine falcon, the spiral of galaxies, and the spiral of mollusk shells.²⁶ Phi was found to hold such aesthetic value that it served as the basis for Western art and architecture.²⁷ (There is a debate as to how consciously the number was applied, especially in ancient art and architecture where close approximations of the proportion are found to be used by peoples who may not have known specifically of the number.)²⁸

Finally, a rough approximation of phi known as the "rule of thirds" has been adopted by photographers.²⁹ According to this rule, the composition of a photograph is more aesthetically pleasing if the subject/s is/are placed off-center at lines dividing the canvas into thirds.

This approximation reminds us of Rav's ratio of one-third *gdil* to two-thirds *anaf* for aesthetically pleasing *tsitsit*! This, I propose, is the resolution to our halakhic quandary as to how the law can require us to have beautiful *tsitsit* while dictating Rav's definition of "beautiful." Applying what we have learned above about aesthetics and phi, we see that Rav's definition is not merely an expression of his own personal taste but rather an echo of a universal chord,³⁰ or even of objective beauty found in creation itself.³¹

Approximations

Regarding the imprecision of Rav's ratio with respect to the number phi, the approximation of thirds (i.e., 1.5)³² simply provides a close enough approximation depending on context, as demonstrated by the rule of thirds in photography. Similarly, the nature of *tsitsit* is such that they are produced in the rough and observed in the rough. Even meeting this ratio precisely appeared too demanding for Hazon Ish, who asked how the Sages could require an exact ratio. His brother-in-law, Rabbi Yaakov

Yisrael Kanievsky, known as the Steipler Gaon, replied in his "Measurements in the Torah" that Rav's ratio is not meant to be met precisely but rather in approximation by the naked eye because beauty is beheld by the naked eye.³³

Indeed, while phi figures in much of nature on a precise mathematical basis, it is found in much of art and architecture in approximation. The argument between those who consider phi to be the basis for works of art versus those who hold such to be wishful thinking centers on whether phi is used precisely or not. However, in all the cases of contention, even phi detractors agree that, whether created consciously or otherwise, approximations to phi exist. Thus, there is consensus that many works of art and architecture successfully use proportions varying within a range of ± 10 percent of the Golden Ratio (i.e., 1.456 to 1.780).³⁴ It can be said, then, that phi need not be met to the angstrom in order to bequeath the beauty it manifests but rather it need only be met in rough approximation to provide aesthetically pleasing proportion.

Ultimately, proper proportion is the most highly sought element in aesthetics. In the words of the thirteenth-century Italian philosopher Thomas Aquinas: "The senses delight in things duly proportioned" (*Summa Theologica* I.5.4).

Known to have argued for "simple proportion" using the ratio of two whole numbers (e.g., $3/2$),³⁵ the first-century Roman architect Marcus Vitruvius Pollo said, "A magnificent temple cannot be constructed properly, unless it is built in an orderly manner with regard to symmetry and proportion of its parts, as is the case with a well-built man..." (*De Architectura Libri Decem*).

Rav's intent was to define an objectively pleasing proportion that was at the same time easy to implement.³⁶ By stipulating "one-third, two-thirds," his rule is easy to apply while still remaining close enough to express the natural beauty of the Divine proportion.

Beauty in Halakhah

Beyond the question of subjective versus objective beauty (i.e., how can halakhah presume to “define” beauty), there is a teleological question. Given that Judaism in general, and halakhah in specific, is concerned with ultimate goals and ultimate purpose, how can halakhah engage in something so seemingly ephemeral as aesthetics? Why should halakhah demand beauty in *tsitsit*?

In his study on the human condition, Rabbi Joseph B. Soloveitchik provides an answer to the teleological question.³⁷ Rabbi Soloveitchik divides human experience into three “gestures”: intellectual, ethical, and aesthetic.³⁸ The intellectual and ethical are teleological undertakings in which a person endeavors to reach absolute truths, be they of nature or of morality.³⁹ Aesthetic undertakings, on the other hand, are not necessarily so animated. Within the *superficial* aesthetic experience, the aesthete simply strives after subjective pleasure, having no greater goal than to satisfy the self.⁴⁰ When pleasure is the highest goal, sin is the inevitable result.⁴¹ It can be said then, that the aesthetic is ultimately at the root of all sin. Indeed, it was the source of the primordial sin, Eve having decided to eat from the forbidden tree because of its aesthetic allure (Genesis 3:6).

Rabbi Soloveitchik explains that “What caused man’s fall is his giving preference to the sensuous, delightful, and pleasing over the true, at both the intellectual and ethical levels.”⁴² That is, on the intellectual level man was told by G-d, his Creator and source of wisdom, “on the day that you eat it you will die” (Genesis 2:17); and on the ethical level man was *commanded* by G-d, his Creator and moral arbiter, “from the tree of knowledge of good and evil do not eat” (Genesis 2:17). Nevertheless, Eve disregarded these considerations in favor of the aesthetic, observing that “the tree was good for food and desirable to the eyes” (Genesis 3:6).

Perhaps Eve’s mistake is the basis for the severity of the Mishnah, “One who walks by the way and breaks off his study and says, ‘How beautiful is this tree...’ makes his life forfeit” (Mishnah *Avot* 3:7). For if, in the midst of the intellectual/ethical gesture, a person stops to satisfy the aesthetic,⁴³ he performs an act akin to that of Eve’s, which eventuated in death, and thus he “makes his life forfeit.” Interestingly, in both the Garden of Eden

account and the example in *Avot* 3:7, the source of natural beauty is a tree.

The aesthetic gesture, however, is not unredeemable; on the contrary, it is essential that humankind harness it to serve G-d.⁴⁴ And so the same tree for which one made his life forfeit by gazing at in self-indulgence, or at the expense of the intellectual/ethical,⁴⁵ can also be the subject of a special blessing when it is appropriately recognized as a gift from G-d.⁴⁶ Indeed, a blessing elevates an act from one of selfish pleasure to one of appreciation of, and communion with, the transcendental. Rabbi Soloveitchik explains that for the aesthetic experience to be meaningful, it must aspire to the absolute, to the transcendent: “[It] must always be encountered as a reflection of Divine beauty.”⁴⁷

Redemption of the aesthetic gesture is accomplished by applying the same teleological aspirations that make the intellectual and ethical gestures meaningful. “When the aesthete begins to wonder whether everything which is apprehended as beauty and as pleasant expresses indeed genuine beauty, when he thinks that the aesthetic act can be critically examined and its worth objectively ascertained, in a manner similar to our critical attitude toward cognitive and ethical gestures, then beauty is redeemed.”⁴⁸

As such, just as the intellectual and ethical quests lead to G-d as the ultimate source of truth and goodness, so too does the *profound* aesthetic experience lead to G-d as the ultimate source of beauty. “G-d not only addresses Himself to man through the logos, by emanating wisdom and knowledge to the finite mind; not only through the ethos—revealing to natural man, driven by insensate desires and impulses, a great order of absolute values and ideals—but also through aesthesis—the immediate sensible apprehension of reality which is beautiful and grandiose.”⁴⁹

And in so reaching G-d through the aesthetic, a person can realize an ecstatic relationship unattainable through the other gestures. “Only through coming in contact with the beautiful and exalted may one apprehend G-d instead of comprehend Him...”⁵⁰

This, then, answers the question of why halakhah should be concerned with the aesthetic. The aesthetic gesture holds great attraction and is

highly prone to abuse. Indeed, it is the very source of man's downfall.⁵¹ Thus, halakhah, which comes to direct our every action to do what is right and good, must also direct us to apprehend G-d through the aesthetic as well.⁵² In so doing, halakhah trains us to attune our aesthetic impulse to the transcendent, where we will find ultimate fulfillment.

As such, though there may be things that one does not find particularly pleasing, if they are said to obtain of aesthetic truth, one must yield to them, knowing that one is thereby redeeming the aesthetic gesture within oneself. Correspondingly, explains Rabbi Soloveitchik, "One must be able to reject certain phenomena even though they please the eye, as 'false' manifestations of true beauty."⁵³ In this regard, Rav's ratio must be adopted irrespective of whether it appeals to our personal taste or not.

Although I have attempted to demonstrate a basis for Rav's ratio in its approximation to phi, Rav's ratio must be maintained as an expression of "true beauty" regardless of our discussion. Perhaps one of Rav's goals in defining beauty was to impart the notion that just as objectivity is essential in ethical judgment, objectivity must also be a part of aesthetic judgment—as our thinking in the one informs our thinking in the other.⁵⁴

Conclusion

The aesthetic, while fraught with potential for abuse, is not only a valid "gesture" but one essential for spiritual self-realization. A person should develop a genuine appreciation of the beautiful as a pathway to encountering the Divine; and at the same time "redeem" the aesthetic experience by seeing it as a reflection of the Divine. In this regard we have seen that, though beauty is in the eye of the beholder, there are things in nature which can be said to have universal, or objective, beauty. Rav's ratio embodies universal, or objective, beauty which correspondingly so endows tekhelet⁵⁵ tsitsit. Thus his ratio adds yet another dimension to the device of the tsitsit that reminds us of the Creator⁵⁶—the Creator of beauty itself.⁵⁷

May this appreciation of beauty serve to inspire yet more Jews to wear tekhelet tsitsit and thus merit the enjoyment of Divine beauty, both physically and spiritually. Regarding the physical reward, Rav Huna teaches

that: “one who is punctilious in the observance of the mitzvah of tsitsit merits a beautiful *talit* (prayer shawl).”⁵⁸ And regarding the spiritual reward, Rabbi Shimon bar Yoḥai promises that: “one who is punctilious in the observance of the mitzvah of tsitsit merits seeing the Divine Presence.”⁵⁹

Acknowledgment

I dedicate this article to my good friend Joel Guberman, founder of the Ptil Tekhelet Association, without whose ongoing support and friendship this article would not have been possible.

Notes

- ¹ The term “tekhelet” here is used to refer to the entire tsitsit tassel and not specifically to the tekhelet strings. See Rashi on Talmud *Menahot* 39a, s.v. tekhelet.
- ² As taught by Rabi in a *baraita* (*Menahot* 39a). See Rashi on *Menahot* 39a, s.v. *ela hulyah*.
- ³ Braiding can take up the majority of the tassel, but not its entirety (Magen Avraham, *Orah Hayyim* 11:14:20).
- ⁴ <http://www.tekhelet.com/pdf/ShneiShlish.pdf>
- ⁵ Perhaps Maimonides’ position of requiring Rav’s ratio only when employing tekhelet strings is based on the use of the term “tekhelet” in *Menahot* 39a specifically referring to the entire tassel, thus implying that the rule is only applicable when tekhelet dye is used. Alternatively, Maimonides’ position might be based on the fact that the contrast between *anaf* and *gdil* is only pronounced enough to have an aesthetic impact when the *gdil* comprises blue strings. Although the *Shulhan Arukh* does not quote Maimonides’ position, nevertheless perhaps this is why the ratio is not strictly adhered to today, since people have become accustomed to using only white strings, wherein the ratio lacks the impact rendered by the inclusion of tekhelet-colored strings.
- ⁶ See also Beit Yosef, *Orah Hayyim* 11:4.
- ⁷ Also Ba’er Hetev, *Orah Hayyim* 11:22.
- ⁸ *Mishnah Brurah*, *Orah Hayyim* 11:66, notes that the obligation is rabbinic.
- ⁹ Examples of such circumstances are “erev Shabbat” (*Mishnah Brurah*, *Orah Hayyim* 11:66) and being “on the road” (Biur Halakhah, *Orah Hayyim* 11:4, s.v. *yoter*).
- ¹⁰ Modifying existing tassels is not always possible because reducing the tassel length might bring the tsitsit under the minimum four and eight thumb-breadth requirements; similarly, removing or adding a *hulyah* might bring the total under the seven *hulyot* or over the thirteen *hulyot* requirement range (Talmud *Menahot* 39a).
- ¹¹ Gordon Thomas, “Are Aesthetic Judgments Subjective or Objective?” <http://www.oswestryschool.org.uk/decimus/aesthet.htm>.
- ¹² *Ibid.*
- ¹³ Avowed subjectivist Kant would say: The object of which is universally *perceived* to admit of some quality.
- ¹⁴ John Hyman, “Is Beauty in the Eye of the Beholder.” <http://www.oswestryschool.org.uk/decimus/beauty.htm>.
- ¹⁵ Immanuel Kant, *The Critique of Judgment* (1790), part I: “Critique of the Aesthetic Judgment,” chap. 45.
- ¹⁶ Though Kant argues that a work of art seems unfettered by rules, his point is not that no rules should be applied. Rather, he argues, the application of rules should be indiscernible.
- ¹⁷ For a treatment of the number phi see Mario Livio, *The Golden Ratio: The Story of Phi, the World’s Most Astonishing Number* (NY: Broadway Books, 2002).
- ¹⁸ *Ibid.*, p. 10. It should be noted that Livio uses the word “enduring” to mean that phi has persisted

throughout history to be viewed as a canon, but not that he himself believes it to be such.

- ¹⁹ It is conjectured that the Pythagoreans in the fifth century BCE knew of phi (Livio, pp. 34-41).
- ²⁰ "A straight line is said to have been cut in extreme and mean ratio when, as the whole line is to the greater segment, so is the greater to the lesser" (Euclid, *Elements*, Book VI, Definition III). See also Livio, pp. 75-78. As an interesting side note, the eighteenth-century Lithuanian scholar Rabbi Eliyah ben Shlomoh Zalman (the Vilna Gaon) said to the translator of Euclid's work into Hebrew: "To the degree that a man is lacking in the wisdom of mathematics he will lack one hundredfold in the wisdom of the Torah" (cited in Joseph B. Soloveitchik, *Halakhic Man*, [Jerusalem: Sefer ve Sefel, 2005] p. 57).
- ²¹ Livio, p. 118.
- ²² *Ibid.*, p. 116.
- ²³ An interactive sunflower growth simulation is available at: http://cinderella.de:80/files/HTMLDemos/7S04_Sunflower.html, which demonstrates the efficiency of seed packing using the Golden Angle versus other similar angles.
- ²⁴ For example: the pentagram, Platonic solids, fractal geometry, and certain crystal structures.
- ²⁵ Luca Pacioli wrote a three-volume work on the subject of phi in 1509 entitled *De Divina Proportione*.
- ²⁶ Interestingly, the shell of the *Murex trunculus*, the snail from which tekhelet dye is made, appears to follow the phi-based logarithmic spiral.
- ²⁷ Artists who applied the phi proportions include Leonardo da Vinci (at least for the artwork in Pacioli's book), Paul Serusier, Giro Severini, Juan Gris, Jacques Villon, Le Corbusier, and Salvador Dali. Jacques Villon enthusiastically pronounced, "As in the Middle Ages one told a prayer before beginning a painting, I rely on the golden section for the surety of ancient times."
- ²⁸ For example, the Pyramids and the Parthenon (see Livio, pp. 51-61, 72-75).
- ²⁹ In photography 1.5 is used instead of 1.618. Dividing a line into thirds gives a ratio of $(1 + s) / 1 = 3:2$. Phi, however, gives 1.618:1 or 3.236:2. See: http://www.istockphoto.com/article_view_print.php?ID=13; http://www.thinctanc.co.uk/design/photography_tutorial_02.html; http://www.markboulton.co.uk/articles/detail/feeling_your_way_around_grids/; http://photoinf.com/Golden_Mean/Edwin_Leong/Camera_Hobby_-_e-Book_on_the_Golden_Ratio.htm; http://en.wikipedia.org/wiki/Rule_of_thirds
- ³⁰ Philosophers David Hume and Immanuel Kant would argue that though we cannot claim an object to be inherently beautiful, we can, as Man, claim that the object is beautiful for all Man.
- ³¹ Plato held that beauty exists objectively and that concrete things can participate in that objective beauty.
- ³² See note 29.
- ³³ Cited by Taitelbaum, p. 3. See note 4.
- ³⁴ See, for example, the discussion on McManus in Livio, p. 182.
- ³⁵ Livio, p. 161.
- ³⁶ Tsitsit are ritual objects that must be produced with relative ease because they undergo the tremendous wear and tear of daily use and thus must be replaced relatively often. This is as opposed to tefillin that require a more exacting standard of manufacture (i.e., the boxes must be a perfect square, see *Shulhan Arukh, Oraḥ Hayyim* 32:39). Tefillin are produced by experts and built to last a lifetime; nevertheless, even here it is understood that absolute precision is not attainable (see *Mishnah Brurah, Oraḥ Hayyim* 32:39:176).
- ³⁷ Joseph B. Soloveitchik, *Worship of the Heart* (NJ: Ktav, 2003), "The Human Condition and Prayer," pp. 37-50 and "Exaltation and Redeeming the Aesthetic," pp. 51-72.
- ³⁸ *Ibid.*, pp. 40-42.
- ³⁹ *Ibid.*, pp. 42, 53.
- ⁴⁰ *Ibid.*, p. 43.
- ⁴¹ *Ibid.*, p. 46.
- ⁴² *Ibid.*, p.47. Similarly, Rabbi Samson Raphael Hirsch on Genesis 2:16.
- ⁴³ Bartenura on *Avot* 3:7 comments that even when one gazes at the tree in appreciation of G-d's handiwork, one is nonetheless culpable for stopping his engagement in the intellectual/ethical. This view highlights Rabbi Soloveitchik's explanation of the original sin in which man placed the sensuous over the intellectual/ethical. Not only is it unacceptable to pursue the sensuous in and of itself, but even when one seeks God through the aesthetic, such a gesture cannot come at the expense of the intellectual/ethical.

- ⁴⁴ Soloveitchik, pp. 51-64. Indeed, the *yetser ha'ra* is seen by the Talmud Sages as a drive, on the one hand capable of bringing man's downfall, yet on the other hand, essential for man to actualize his potential. (See Rabbi Chaim Eisen, "You Will Be Like G-d," *Jewish Thought*, vol. 2, no. 1, pp. 45-116.
- ⁴⁵ See note 43.
- ⁴⁶ *Shulhan Arukh, Oraḥ Hayyim* 225:10.
- ⁴⁷ Soloveitchik, p. 52.
- ⁴⁸ *Ibid.*, p. 56.
- ⁴⁹ *Ibid.*, p. 57.
- ⁵⁰ *Ibid.*, p. 59.
- ⁵¹ Rabbi Soloveitchik (*ibid.*, p. 48) explains that as a result of the original sin based on placing the aesthetic above the intellectual and ethical, Man became ashamed of his nudity precisely due to his aesthetic awareness. In this sense, clothing comes to address the abuse of the aesthetic experience. As such, clothing is the ultimate reminder of the original sin. Indeed, the *Zohar on Ki Teitseh* 276a notes that *beḡed*, clothing, has the same root letters as *boged*, unfaithful. In consonance, one of the purposes of the *tsitsit* attached to the corners of our clothes is to remind us not to stray after our eyes.
- ⁵² "The task of Halakhah is to sanctify nature" (*ibid.*, p. 143). In *Halakhic Man* (Jerusalem: Sefer ve Sefel, 2005) p. 48, Rabbi Soloveitchik explains that halakhic man finds G-d not in the supernal realms but rather in the very midst of our concrete reality: "...the Divine Presence in dimensions and the glory of G-d in measurements."
- ⁵³ Soloveitchik, *Worship of the Heart*, p. 54.
- ⁵⁴ See Sam Fleischacker, *A Third Concept of Liberty* (Princeton University Press, 1999) chap. 2. See also my paper "Halacha, Ethics and Aesthetics," *Everett Journal of Jewish Ethics* (Winter 2008) vol. 1.
- ⁵⁵ As noted earlier, Rav's statement refers to *tsitsit* explicitly as "tekhelet," and, as indicated by Maimonides, the ratio is most readily noticed when tekhelet provides the necessary contrast.
- ⁵⁶ Numbers 15:39. "Upon seeing the tekhelet strand one remembers his Creator" (Midrash *Tanḥuma, Shelah* 15), and also Rashi on Talmud *Menahot* 43b (s.v. *v'raḳia*). Recanati's commentary on Numbers 15:37-40 quotes the explanation of Bahir (92-93) that the thirty-two *tsitsit* strands are a symbolic reminder of the "ways" of the King, and the tekhelet is a symbol of the King Himself. Rabbi Isaac Luria says that tekhelet is the symbol of G-d's Kingship (*Pri Ets Hayyim, Shaar Ha'Tsitsit*, chap. 4). See also *Sifri, Shelah* 115; *Pesikta Zutra (Lekah Tov, Shelah* 112b); *Midrash Aggadah* ([Buber] Numbers 15:38); *Yalkut Shimoni (Shelah, remez* 750); *Jerusalem Talmud Brakhot*, chap. 1: 3a). "Looking at the tekhelet is as if seeing the Divine Presence" (*Mishnat Rabbi Eliezer*, chap. 14, p. 264). "When Israel looks upon the tekhelet strand it appears to them as if the Divine presence is amongst them" (*Tanḥuma* [Buber] *Shelah* 30). Similarly, *Midrash Tehillim* ([Buber] 90:18); *Yalkut Shimoni (Tehillim, remez* 841). See also *Menahot* 43b.
- ⁵⁷ Or, according to Kant: the Creator of the human ability to perceive beauty.
- ⁵⁸ Talmud *Shabbat* 23b.
- ⁵⁹ Talmud *Menahot* 43b.

אמר רב: תכלת שכרך רובה בשרה
 ואפילו לא כרך בה אלא חוליא אחת בשרה
 ונויי תכלת שלישי גדיל ושני שלישי ענף