

Pascal's Wager

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..והוי מחשב הפסד מצוה כנגד שכרה ושכר עבירה כנגד הפסדה" (אבות ב:א).

Excerpts from:

Hájek, Alan, "Pascal's Wager", *The Stanford Encyclopedia of Philosophy (Spring 2004 Edition)*, Edward N. Zalta (ed.),
<http://plato.stanford.edu/archives/spr2004/entries/pascal-wager/>

We find in it the extraordinary confluence of several important strands of thought: the justification of theism; probability theory and decision theory, used here for almost the first time in history; pragmatism; voluntarism (the thesis that belief is a matter of the will); and the use of the concept of infinity.

Pascal is apparently unimpressed by such attempted justifications of theism: "Endeavour ... to convince yourself, not by increase of proofs of God..." Indeed, he concedes that "we do not know if He is ...". Pascal's project, then, is radically different: he seeks to provide *prudential* reasons for believing in God. To put it crudely, we should wager that God exists because it is the *best bet*.

Decision under Uncertainty

No probabilities.

Two possible actions, A1 and A2, the worst outcome associated with A1 is at least as good as the best outcome associated with A2; also, in at least one state of the world, A1's outcome is strictly better than A2's.

So A1 *superdominates* A2.

Rationality requires you to perform A1.

Decision under Risk

Example: Suppose that the utility of money is linear in number of dollars: you value money at exactly its face value. Suppose that you have the option of paying a dollar to play a game in which there is an equal chance of returning nothing, and returning three dollars. The expectation of the game itself is

$$0 \cdot (1/2) + 3 \cdot (1/2) = 1.5,$$

so the expectation of paying a dollar for certain, then playing, is

$$-1 + 1.5 = 0.5.$$

This exceeds the expectation of not playing (namely 0), so you should play. This is known as the argument from "utility" or "expectation".

Pascal's Argument from Superdominance

"God is, or He is not." But to which side shall we incline? Reason can decide nothing here. There is an infinite chaos which separated us. A game is being played at the extremity of this infinite distance where heads or tails will turn up... Which will you choose then? Let us see. Since you must choose, let us see which interests you least. You have two things to lose, the true and the good; and two things to stake, your reason and your will, your knowledge and your happiness; and your nature has two things to shun, error and misery. Your reason is no more shocked in choosing one rather than the other, since you must of necessity choose... But your happiness? Let us weigh the gain and the loss in wagering that God is... If you gain, you gain all; if you lose, you lose nothing. Wager, then, without hesitation that He is.

Basically, Pascal is here asserting here that there is no down side for acting like there is a God, and on the other hand there is a loss for acting like there is no God.

Some have rightly attacked the argument from super-dominance on the basis that one does lose "the true". That is, if one wagered for God and there turned out to be no God, then clearly one lived a lie. This, one could argue, is a fairly inconsequential result given that such a "lie" didn't have any great negative consequences, one still living a happy life.

Jews cannot easily accept the argument from super-dominance because there is clearly a "loss" in that performance of the 613 cause one to quite demonstrably (and designedly) modify behavior, to which one might claim, besides monetary costs, a "cramping" of one's lifestyle. Nevertheless, as Pirke Avot (2:1) explains, if you will carefully weigh the reward for religious observance versus its cost and the loss for lack of observance against its gain, you will come to realize that its religious observance is worthwhile.

Pascal's Argument from Expectation

Let us see. Since there is an equal risk of gain and of loss, if you had only to gain two lives, instead of one, you might still wager. But if there were three lives to gain, you would have to play (since you are under the necessity of playing), and you would be imprudent, when you are forced to play, not to chance your life to gain three at a game where there is an equal risk of loss and gain. But there is an eternity of life and happiness.

But there is an eternity of life and happiness. And this being so, if there were an infinity of chances, of which one only would be for you, you would still be right in wagering one to win two, and you would act stupidly, being obliged to play, by refusing to stake one life against three at a game in which out of an infinity of chances there is one for you, if there were an infinity of an infinitely happy life to gain. But there is here an infinity of an infinitely happy life to gain, a chance of gain against a finite number of chances of loss, and what you stake is finite. It is all divided; wherever the infinite is and there is not an infinity of chances of loss against that of gain, there is no time to hesitate, you must give all...

	<i>God exists</i>	<i>God does not exist</i>
<i>Wager for God</i>	∞	f_1
<i>Wager against God</i>	f_2	f_3

$$E(\text{wager for God}) = \infty * p + f_1 * (1 - p) = \infty$$

$$E(\text{wager against God}) = f_2 * p + f_3 * (1 - p)$$